DEVELOPPEMENTS LIMITÉS

Formule de Taylor-Young en 0 :

$x^n \varepsilon(x)$ est le reste

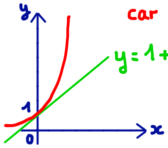
$\lim_{x \to 0} \varepsilon(x) = 0$

$$f(x) = f(0) + f'(0)\,x + f''(0)\frac{x^2}{2!} + \dots + f^{(n)}(0)\frac{x^n}{n!} + x^n \varepsilon(x)$$

$f(0) + f'(0)\,x$: tangente en 0

Le terme non nul suivant donne la position relative locale

e^x au dessus car $\frac{x^2}{2} \geq 0$



DL usuels en 0 :

car $e^0 = 1$ et $(e^x)' = e^x$

$$e^x = 1 + x + \frac{x^2}{2!} + \dots \frac{x^n}{n!} + x^n \varepsilon(x)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + x^{2n}\varepsilon(x)$$ puissances paires

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + x^{2n+1}\varepsilon(x)$$ puissances impaires

} SIGNES ALTERNÉS

$$\text{ch}\,x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + x^{2n}\varepsilon(x)$$ ← car $\text{ch}\,0 = 1$, $\text{sh}\,0 = 0$, $(\text{ch}\,x)' = \text{sh}\,x$, $(\text{sh}\,x)' = \text{ch}\,x$

$$\text{sh}\,x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + x^{2n+1}\varepsilon(x)$$

$\text{ch}\,x = \frac{e^x + e^{-x}}{2}$ $\quad$ $\text{sh}\,x = \frac{e^x - e^{-x}}{2}$

$$(1+x)^m = 1 + mx + m(m-1)\frac{x^2}{2!} + \dots + m(m-1)(m-2)\cdots(m-n+1)\frac{x^n}{n!} + x^n\varepsilon(x)$$

poser $m = -1$

$$\frac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots + (-1)^n x^n + x^n \varepsilon(x)$$

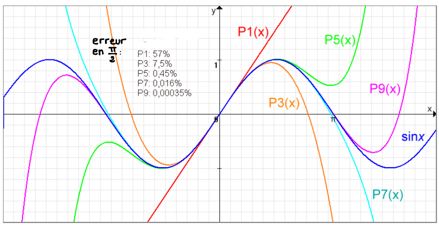
poser $m = \frac{1}{2}$

$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + x^2\varepsilon(x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + x^5\varepsilon(x)$$ (intégrer $\frac{1}{1+x}$)

$$\text{Arctan}\,x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + x^7\varepsilon(x)$$ (intégrer $\frac{1}{1+x^2}$)

Approximations de sinx par des polynômes



$P_1(x) = x$

$P_3(x) = x - \frac{x^3}{6}$

$P_5(x) = x - \frac{x^3}{6} + \frac{x^5}{120}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^4\varepsilon(x)$$ ← poser $X = -x \to 0$ si $x \to 0$ dans $\frac{1}{1+X}$